Covering Property Axiom CPA,
a combinatorial core of the iterated perfect set model
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Many interesting mathematical properties, especially concerning real analysis, are known to be true in the iterated perfect set (Sacks) model, while they are false under the continuum hypothesis. However, the usual proofs that these facts are indeed true in this model are very technical and involve heavy forcing machinery. The work that I will report in my presentation is changing this state of things.

In the talk I will formulate a combinatorial axiom principle CPA, that is true in the model, and demonstrate how to use it. In particular, I will show how CPA implies the following statements.

- For every subset S of $\mathbf{R}$ of cardinality 2^ω there exists a (uniformly) continuous function $f:\mathbf{R}\to[0,1]$ such that $f[S]=[0,1]$.
- Every perfectly meager set $S\subset\mathbf{R}$ has cardinality less than 2^ω.
- Every universally null set $S\subset\mathbf{R}$ has cardinality less than 2^ω.
- The cofinality of the measure ideal is less than 2^ω.
- There exists a family $\mathcal{F}\subset[\omega]^\omega$ of cardinality less than 2^ω which is maximal almost disjoint, MAD.
- The plane $\mathbf{R}^2$ can be covered by less than 2^ω many sets each of which is a graph of a continuously differentiable function from $\mathbf{R}$ into $\mathbf{R}$ in either horizontal or vertical axis.
- There exists a family $\mathcal{H}$ of less than 2^ω pairwise disjoint perfect sets such that $\bigcup\mathcal{H}$ is a linear basis of $\mathbf{R}$ over $\mathbf{Q}$.
- There exists a non-principal selective ultrafilter on ω.

The axiom will be presented in stages, starting from the simplest form, which implies already first four of these statements.